HEAT TRANSFER ACROSS VERTICAL FERROFLUID LAYERS

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Abstract—The numerical and experimental study is presented of convective heat transfer in a vertical layer of a ferromagnetic fluid. An effect is investigated of the direction of a horizontal gradient of magnetic field strength relative to that of the temperature gradient on convective motion. A criterial relationship is given between heat transfer and characteristic parameters.

NOMENCLATURE $\ddot{}$

$$
x_2^{(\pm m)}, = x_\alpha \pm mh, \hat{t} = t + \tau/2, \check{t} = t - \tau/2,
$$

coordinates of grid pattern points with
center at node (x_1, x_2, t) ;

$$
u^{(\pm m_1)}, = u(x_1^{(\pm m)}, x_2, t), u^{(\pm m_2)} = u(x_1, x_2^{(\pm m)}, t),
$$

$$
\check{u} = u(x_1, x_2, \check{t}), \hat{u} = u(x_1, x_2, \hat{t}),
$$
 designations for grid functions;
 c_1 , = const ≥ 0 .

1. INTRODUCTION

CONVECTION and heat transfer in closed volumes in a gravitational field is of interest when evaluating heatinsulating properties of fluid layers and when calculating heat transferred from one of the walls of a heated chamber. A great deal of theoretical and experimental works are concerned with the laws of free convection in cavities with different heat-transfer agents. Synthesis of ferrofluids and systematic study of their properties $\lceil 1-4 \rceil$ offer new possibilities for solution of the problems mentioned above. The fact, that a unit volume of a ferromagnetic fluid possesses its own magnetic moment makes it possible to control the heat-transfer coefficient by applying a magnetic field.

The present paper is aimed at theoretical and experimental study of convective heat transfer across vertical layers of a ferrofluid heated from one side and placed in a gradient magnetic field (Fig. 1).

If we neglect magnetocaloric and magnetostrictive the form effects and then make usual Boussinesq approximations, a system of the Rosensweig-Neuringer ferro-
 $F = F(x_1) = L(\xi_0) + A_1 \xi_0 \frac{F(x_1 - x_2)}{\lambda \xi_0}$, (15) hydrodynamics equations for a non-conducting incompressible ferrofluid may be written in the form $[1, 3]$

$$
\frac{DT}{Dt} = \kappa \nabla^2 T \tag{1}
$$

$$
\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2 \mathbf{v} + \frac{1}{\rho_0}\rho \mathbf{g} + \frac{\mu_0}{\rho_0}\mathbf{M}\nabla H \qquad (2)
$$

$$
\mathbf{v}\,\mathbf{v} = 0\tag{3}
$$

$$
rot H = 0, \ \ div(H + M) = 0, \ \ M = \frac{M}{H}H \tag{4}
$$

$$
\rho = \rho_0 [1 - \beta (T - T_0)], \ \ M = M(T, H) \tag{5}
$$

with the following boundary conditions $\psi'(0, x'_2, t') = \psi'(1, x'_2, t')$

$$
T(0, x_2, t) = T_0 \tag{6} \qquad \qquad = \psi'(x'_1, 0, t') = \psi'\left(x'_1, \frac{a}{t}\right)
$$

$$
T(b, x_2, t) = T_1 > T_0 \tag{7}
$$

$$
T(x_1, 0, t) = T(x_1, a, t) = \frac{\Delta T}{b} x_1 + T_0 \tag{8}
$$

$$
V(0, x_2, t) = V(b, x_2, t)
$$

= $V(x_1, 0, t) = V(x_1, a, t) = 0.$ (9)

For the fluid magnetization the Langevin formula For the holds $[2,4]$
holds $[2,4]$

$$
M = \varepsilon \cdot M_s L(\xi), \ L(\xi) = \coth \xi - \frac{1}{\xi}, \tag{10}
$$

$$
\varepsilon = \mu_0 V_0 M_s \ H \qquad M = M_0 - \lambda (T - T_0)
$$

Neglecting the ferrofluid effect on the properties of the magnetic field, the strength of the latter is held to be a linear function of the spatial coordinates

$$
H = \frac{\Delta H}{b} \left(x_1 - \frac{b}{2} \right) + \tilde{H}, \quad \tilde{H} = 0.5 (H_0 + H_1) \quad (11)
$$

which is chosen for the layer thickness, *6,* of IOmm from the experimental relationship $H = H(x_1)$.

The following dimensionless variables are introduced

$$
x'_{1} = \frac{x_{1}}{b}, \ x'_{2} = \frac{x_{2}}{b}, \ t' = \frac{t}{b^{2}} v, \ V' = \frac{V}{v} b,
$$

$$
M' = \frac{M}{M_{0}}, \ H' = \frac{\mu - \tilde{H}}{\Delta H}, \ T' = \frac{T - T_{0}}{\Delta T}.
$$
 (12)

Eliminating pressure terms by differentiation and subtraction of the moment equations and introducing the In the region of V we set stream function and vorticity yield the following boundary-value problem :

(i) energy equation

$$
\frac{DT'}{Dt'} = \frac{1}{Pr}\nabla^2 T'\n\tag{13}
$$

(ii) vorticity equation

$$
\frac{D\omega'}{Dt'} = \nabla^2 \omega' + Gr \frac{\partial T'}{\partial x_1'} + Gr \cdot A \cdot F \frac{\partial T'}{\partial x_2'},\tag{14}
$$

2. MATHEMATICAL FORMULATION OF THE PROBLEM where the dimensionless function F at $\Delta T \ll T_0$ is of

$$
F = F(x'_1) = L(\xi_0) + A_1 \xi_0 \frac{\partial L(\xi_0)}{\partial \xi_0},
$$
(15)

$$
\xi_0 = \xi_0(x'_1) = A_2 [B(x'_1 - \frac{1}{2}) + 1]
$$

(iii) the stream function equation

$$
\nabla^2 \psi' = -\omega', \ \frac{\partial \psi'}{\partial x'_2} = V'_1, \frac{\partial \psi'}{\partial x'_1} = -V'_2 \tag{16}
$$

(iv) the boundary conditions

$$
\text{div } \mathbf{v} = 0 \tag{17}
$$

$$
T'(1, x'_2, t') = 1 \tag{18}
$$

$$
T'(x'_1, 0, t') = T'\left(x'_1, \frac{a}{b}, t'\right) = x'_1 \tag{19}
$$

$$
= \psi'(x'_1, 0, t') = \psi'\left(x'_1, \frac{a}{b}, t'\right) = 0 \quad (20)
$$

$$
V'(0, x'_2, t') = V'(1, x'_2, t')
$$

=
$$
V'\left(x'_1, \frac{a}{b}, t'\right) = V'(x'_1, 0, t) = 0.
$$
 (21)

3. NUMERICAL ALGORITHM

numerically with the help of the finite-difference method. A difference scheme is built using the integration-interpolation method which provides the $\zeta = \frac{\mu_0 V_0 M_s}{k} \cdot \frac{H}{T}$, $M_s = \frac{M_0 - \lambda (T - T_0)}{k}$. validity of the integral laws of conservation (heat, mass, momentum, energy, etc.) over arbitrary grid sections. momentum, energy, etc.) over arbitrary grid sections.

> The construction of the scheme is illustrated on the following equation of a general form

$$
c_1 \frac{\partial u}{\partial t} = \sum_{\alpha=1}^2 \frac{\partial L_\alpha u}{\partial x_\alpha} + f(x, t), \quad L_\alpha u = \frac{\partial u}{\partial x_\alpha} - V_\alpha(x, t)u. \tag{22}
$$

In a semi-space, $t \ge 0$, take a unit rectangularparallelepiped, V, formed by the planes $x_1 = x_1^{(-0.5)}$, $x_1 = x_1^{(+0.5)}, x_2 = x_2^{(-0.5)}, x_2 = x_2^{(+0.5)}, t = \tilde{t}, t = \hat{t}$. Integrate the both sides of equation (22) with respect to volume V

$$
c_{1} \int_{x_{1}^{(-0.5)}}^{x_{1}^{(+0.5)}} \int_{x_{2}^{(-0.5)}}^{x_{2}^{(+0.5)}} u \Big|_{t=\tilde{t}}^{t=\tilde{t}} dx_{1} dx_{2}
$$

=
$$
\sum_{\alpha=1}^{2} \left(\int_{x_{\beta}^{(-0.5)}}^{x_{\beta}^{(+0.5)}} \int_{\tilde{t}}^{\tilde{t}} L_{\alpha} u \Big|_{x_{\alpha}^{(-x_{2}^{(+0.5)}})}^{x_{\alpha}^{(-x_{3}^{(+0.5)}}} dx_{\beta} dt \right) + \int_{V} f dV. (23)
$$

$$
u\bigg|_{t=\hat{t}} \approx \hat{u}, \quad u\bigg|_{t=\check{t}} \approx \check{u}.
$$
 (24)

Convective and diffusion terms will be approximate just as in the case of steady-state free convection equations of [5]

$$
\frac{D\omega'}{Dt'} = \nabla^2 \omega' + Gr \frac{\partial T'}{\partial x_1'} + Gr \cdot A \cdot F \frac{\partial T'}{\partial x_2'}, \qquad (14) \qquad L_a u \bigg|_{x_x = x_1^{(10.5)}} \approx \Lambda_a^{(10.5_a)} u = \begin{cases} \Lambda_a^+ u^{(10.5_a)}, & \text{if } u^{(10.5_a)} \leq 0\\ \Lambda_a^- u^{(10.5_a)}, & \text{if } u^{(10.5_a)} \geq 0\\ \Lambda_a^- u^{(10.5_a)}, & \text{if } u^{(10.5_a)} \geq 0 \end{cases} \tag{25}
$$

where

$$
\Lambda_{\alpha}^{\pm} u^{(\eta)} = \frac{1}{1 + 0.5|V_{\alpha}^{(\eta)}|h} \cdot \frac{u^{(\eta + 0.5_{\alpha})} - u^{(\eta - 0.5_{\alpha})}}{h}
$$

$$
-V_{\alpha}^{(\eta)} u^{(\eta \pm 0.5_{\alpha})}, \quad \eta = \pm 0.5_{\alpha},
$$

$$
V_{\alpha}^{(\pm 0.5_{\alpha})} = 0.5(V_{\alpha} + V_{\alpha}^{(\pm 1_{\alpha})}), \quad \alpha = 1, 2.
$$

Designate

$$
\Lambda_{\alpha} u = (\Lambda_{\alpha}^{(+0.5_{\alpha})} u - \Lambda_{\alpha}^{(-0.5_{\alpha})} u)/h. \tag{26}
$$

Taking into account (24) – (26) , this allows the difference balance equation for unit cell to be obtained from (23)

$$
c_1 h^2(\hat{u} - \check{u}) = \tau h^2 \sum_{\alpha=1}^2 \Lambda_\alpha u + \int_V f \, dV. \tag{27}
$$

Upon termwise summation of (27) over the points of however great grid region, only algebraic sums of the unknowns are left along the region boundary, which reveals a conservative nature of the scheme (27). Next (27) is written down as a scheme of alternating directions

$$
c_1 \frac{u - \check{u}}{\tau/2} = \Lambda_1 u + \Lambda_2 \check{u} + \frac{1}{\tau h^2} \int_V f \, \mathrm{d}V. \tag{28}
$$

$$
c_1 \frac{\hat{u} - u}{\tau/2} = \Lambda_1 u + \Lambda_2 \hat{u} + \frac{1}{\tau h^2} \int_V f \, dV. \tag{29}
$$

This scheme approximates equation (22) within the error of $\theta(\tau^2 + h^2)$. In [6] proof is presented of the theorem on absolute stability of the method for $\Lambda_{\alpha} = \Lambda_{\alpha}(x_1, x_2).$

Differential equations are constructed according to schemes (28)-(29). The derivatives entering into the coefficients and free terms are approximated by central differences. To realize the scheme use is made of the scalar elimination. Vorticity on the region boundary is roughly determined from the two-point formula accurate to the second order in h [7]. Calculations are performed on a uniform spatial grid with $h = 1/20$.

4. EXPERIMENTAL INSTALLATION

A vertical layer of a ferrofluid with the height a, of 270 mm, thickness b , of 10 mm and width c, of 70 mm is enclosed between two brass plates, side walls and horizontal end inserts made of organic glass (Fig. 2). One of the plates was cooled with thermostated water pumped through a heat exchanger (water discharge, $3 \cdot 10^{-5}$ m³/s). The other plate was heated with an electric Nichrome wire heater (resistance, 750 Ω). To measure heat input, ammeter and voltmeter were used (grade of fit, 0.5). During the experiments power of the heater ranged from $6 \cdot 10^{-1}$ to 10^2 W. The voltage varied in such a way that a Rayleigh number of the steady-state regime could differ from that of the previous regime by $6-10\%$. Local Nusselt numbers were calculated from temperature gradients in a wall ferrofluid layer in the direction normal to the plate surface. Wall temperature was controlled with 14 copper-constantan thermocouples (wire diameter, 0.1 mm) peened into the layer boundaries. Deviations from the mean wall temperature did not exceed 0.1°C. To measure a temperature field in the cavity, a

FIG. 2. Schematic drawing of experimental unit: 1, scale; 2, operating volume of cavity; 3,4, cavity walls; 5, electric heater; 6, foam-plastic insulation; 7, electric motor; 8, micrometric screw; 9, inlet connection of cooler; 10, movable thermocouple probe; 11, movable rule; 12, side walls of cavity; 13, cooler.

5-thermocouple probe was used (wires of 0.05mm in dia) positioned uniformly over the cavity width and moved at a constant speed of 8mm/min by a micrometer screw attached to an electric motor. Drift of thermocouples was equal to 0.2 s. Results of measurements were automatically registered by a two-coordinated recorder, on the x-scanning of which a thermocouple coordinate was read off over the height of the cavity and temperature, along the y-coordinate. A resolving power of the measuring circuit was O.Ol"C. To provide heat insulation, the whole experimental installation was coated with a foam plastic layer 25 mm thick. All the construction units in close proximity to the cavity were made of non-magnetic materials not to distort the imposed magnetic field. The experiments were carried out with a kerosene-base ferrofluid synthesized at the Luikov Heat and Mass Transfer Institute of the Byelorussian Academy of Sciences by magnetite dispersion with oleic acid admixed as a stabilizer. The basic parameters of the fluid are listed in Table 1. Measurements of magnetic and viscous characteristics of the fluid prior to and after the experiments showed negligible "ageing" of the fluid.

Physical properties of the fluid were determined at the layer width-averaged temperature

$\tilde{T} \approx 20^{\circ}$ C.

The cavity was placed in a horizontal magnetic field with approximately linear distribution of *H.* A mean gradient of the magnetic field strength was $\nabla H =$ $3.1 \cdot 10^5$ A/m². Deviations did not exceed 8% .

| Kind of fluid | Øο (kg/m^3) | М (A/m) | ε $\frac{6}{20}$ vol.) | Pr at 20° C | Diameter of particles A | м. A/m www.communication.com |
|------------------|------------------|----------------------------------|---------------------------|-------------------------|----------------------------|--|
| Ferrofluid-3 | 840 | 1027 at $H = 15 \cdot 10^3$ | 1.2 | 36 | $50 - 150$ | $35 \cdot 10^3$ advantagement of the first product in |

Table 1.

5. RESULTS AND DISCUSSION

In calculations the dimensionless criteria varied as follows: $1 \le a/b \le 10$; $10 \le Pr \le 10^3$; $0 \le Ra \le 5 \cdot 10^6$; $0 \le A \le 10^4$; $-1.5 \le B \le 1.5$. Magnetic characteristics of the fluid as well as the cold wall temperature T_0 were not modified but chosen from the experimental conditions. Parameters A_1 and A_2 varied as functions of the magnetic field strength, \tilde{H} , where $0 < \tilde{H} < 2 \cdot 10^5$ A/m .

The outcome of numerical calculations and experimental investigations is detailed information on the effect of the gradient magnetic field, Rayleigh number, cavity sizes on the temperature field structure and integral heat fluxes. Thorough treatment of the calculated and experimental results revealed peculiar features of heat transfer of ferrofluid in the magnetic field. An effect of the geometry parameter, a/b , of the layer on a heat-transfer process was estimated from calculations performed for $a/b = 1, 2, 4, 5, 10$ (Fig. 3). It

FIG. 3. Nusselt number vs cavity dimensions.

appeared that in the region $a/b > 5$ an increase in the geometry parameter results in small decrease of a specific heat flux across the layer at fixed Re and Ra* values. Thus, Nusselt numbers for $a/b = 5$ and $a/b = 10$ differ by not more than 10% (Ra = 3.6·10⁵). Experimental points for $a/b = 27$ verify the possibility for extrapolation comparison. For example, at $Ra^* = 10^6$ the integral Nusselt number predicted for $a/b = 10$ exceeds the experimental value for $a/b = 27$ by $\sim 10\%$ which is within the accuracy of heat measurements made in a free convection situation.

Figure 4 depicts dependence of the heat transferred across the layer on direction of the magnetic strength gradient. Prediction gives three parallel lines with that for $Nu = Nu(Ra)$, when there is no magnetic field, lying between the lines describing the effect of a magnetic field on convective heat transfer in the cavity.

FIG. 4. Comparison of numerical $(a/b = 10)$ and experimental data on heat transfer.

When $\nabla H \uparrow \uparrow \nabla T$, heat-transfer enhancement is observed, but heat-transfer deterioration is observed when $\nabla H \perp \uparrow \nabla T$. From Fig. 4 is seen that up to $Ra \approx 10^6$ the experimental Nu's lie systematically below the predicted ones which appears to result from the error in extrapolation. However, when $Ra \gtrsim 10^6$ the agreement is closer. Now, the sizes of the horizontal boundaries of the cavity in a numerical experiment exert an inverse action on heat transfer. As is known, in the region of $Ra \ge 10^6$ with heat convection in a narrow vertical layer, secondary streams are observed in the form of a vertical row of local vortices on the background of basic convective motion. These secondary structures enhance heat transfer in the layer due to additional heat transfer across the layer on the boundaries of adjacent vortices which is presented by experimental points in Fig. 4. A smaller value of a/b in numerical calculations increases stability of the convective motion in the layer (additional energy consumption for overcoming shear stresses on the horizontal boundaries) and heat transfer preserves the previous rate of enhancement. Temperature profiles at the midheight predicted and measured over the layer width (Fig. 5) correspond to the above dependence of heat transfer on mutual direction of ∇H and ∇T . For one and the same Ra value, a change in the gradient direction of the magnetic field characterizes an increase or decrease of convective heat transfer.

Within the investigated ranges of characteristic parameters no pronounced Nusselt number dependence on Pr was observed, i.e. convective heat transfer proved to be a function of a/b , Ra and Ra*. Correlated results

FIG. 5. Comparison of predicted and experimental temperature profiles in cross-section $x_2 = a/2.$

FIG. 6. An effect of gradient magnetic field on heat transfer $a/b = 5$.

of the integral heat fluxes through the layer are presented in Fig. 6. Horizontal fragments of lines $1-1$; $2-2$ [']; $3-3$ ['] correspond with $Ra^* \ll Ra$, i.e. the case when the magnetic effect is small as compared to the gravitational one and heat transfer is mainly attributed to heat convection. As far as the influence of magnetic forces grows ($Ra^* \approx Ra$), heat transfer either enhances or decreases depending on the direction of the magnetic field. At sufficiently large *Ra** values in the situation $\nabla H \uparrow \downarrow \nabla T$ heat transfer through the layer is effected by conduction and convective component becomes insignificant. In the region $Ra > 10^3$ and $Ra^* > 10^3$, $2 < a/b < 10$ heat transfer in the layer is rather accurately described by criterial relationship

$$
Nu = 0.42(a/b)^{-0.25} [Ra + 4(Ra^*)^{0.91}]^{0.23}.
$$
 (31)

The physical premises of the magnetic field effect on the convective heat transfer in ferrofluid have been widely discussed in literature $[10-13]$, however, the authors are unaware of any quantitative relationships. Criteria1 equation (31) gives vast possibilities for smooth control of heat transfer in a vertical ferrofluid layer in a wide region. For example, changing magnetic parameter Ra^* within $0 < Ra^* < 10^6$ a heat-transfer coefficient may be changed within $1 < Nu < 6$ for $Ra = 10^4$.

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TRANSFERT DE CHALEUR A TRAVERS DES COUCHES VERTICALES DE FLUIDE FERROMAGNETIQUE

Résumé-On présente une étude numérique et expérimentale du transfert de chaleur par convection dans une couche verticale de fluide ferromagnétique. On étudie l'influence sur les mouvements convectifs, de la direction du gradient horizontal de champ magnétique par rapport à celle du gradient de température. Un critère est donné sous la forme d'une relation entre paramètres thermiques et caractéristiques.

DER WARMEUBERGANG IN VERTIKALEN FERROFLUIDSCHICHTEN

Zusammenfassung-Es wird über eine numerische und experimentelle Untersuchung des konvektiven Wärmeübergangs in vertikalen Schichten eines ferromagnetischen Fluides berichtet. Der Einfluß eines horizontalen Gradienten der Magnetfeldstärke auf die Konvektionsbewegung wird im Vergleich zu demjenigen eines Temperaturgradienten untersucht. Eine kriterielle Beziehung zwischen Wärmeübergang und den charakteristischen Parametem wird angegeben.

ИССЛЕДОВАНИЕ ТЕПЛООБМЕНА В ВЕРТИКАЛЬНЫХ СЛОЯХ ФЕРРОМАГНИТНОЙ ЖИДКОСТИ

Аннотация - В работе проведено численное и экспериментальное исследование конвективного теплообмена в вертикальном слое ферромагнитной жидкости. Изучено влияние на конвективное движение направления горизонтального градиента напряженности магнитного поля относительно направления температурного градиента. Приводится критериальная зависимость теплообмена от характерных параметров.

Получено хорошее соответствие результатов эксперимента и расчета.